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Neural Network-based Backcalculation Models for Non-Destructive Evaluation of Rigid Airfield Pavement Systems

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Abstract

Heavy Weight Deflectometer (HWD) tests were routinely conducted on three Portland Cement Concrete (PCC) test items in the Federal Aviation Administration's (FAA's) National Airport Pavement Test Facility (NAPTF) to verify the uniformity of the test pavement structures and to measure pavement responses during full-scale traffic testing. A six-wheel aircraft landing gear with dual-tridem configuration was trafficked on the north side of the test pavement while the south side was trafficked by a four-wheel landing gear with dual tandem configuration. HWD tests were conducted before and during traffic testing at slab center, at longitudinal joints and at transverse joints. Substantial corner cracking occurred in all three of the rigid pavement test items after 28 passes of traffic had been completed. Trafficking continued until the rigid items were deemed failed. This paper presents an Artificial Neural Networks (ANN) based approach for backcalculating the NAPTF rigid pavement properties. The study findings illustrate the complexity of backcalculating properties of rigid pavements subjected to full-scale dynamic traffic testing. Apart from other complicating factors such as the slab curling and warping behavior, the test pavements exhibited corner cracks within few passes of traffic loading which further complicated the interpretation of HWD test results.

Key Words: Neural networks, backcalculation, new generation aircraft, rigid pavements.

Introduction

The National Airport Pavement Test Facility (NAPTF) is located at the Federal Aviation Administration's (FAA) William J. Hughes Technical Center near Atlantic City International Airport, New Jersey, USA. It was constructed to support the development of advanced mechanistic-based airport pavement design procedures based on sound theoretical principles and with models verified from appropriate full-scale test data. The advanced design procedures will take advantage of emerging computational abilities that offer the ability to analyze complex pavement structures and flexibility to evaluate the effects of complex gear configurations under both traffic and environmental loads.

Similar experimental efforts are being carried out under the Airbus A-380 Pavement Experimental Program at Toulouse, France to provide comparative experimental data between different aircraft landing gears, considering especially the Airbus A-380, and to provide full-scale data towards a better understanding of airport pavement behavior (Fabre et al., 2004).

The NAPTF is a fully enclosed state-of-the-art full-scale test facility. The first series of test pavements, referred to as Construction Cycle 1 (CC-1), consisted of nine instrumented test pavements (six flexible and three rigid) that were 18.3 m (60 ft) wide and total 274.3 m (900 ft) in length. The nine test pavements were built on three different subgrade materials: low-strength (target California Bearing Ratio [CBR] of 4), medium-strength (target CBR of

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8), and high-strength (target CBR of 20). A plan view of the CC-1 NAPTF test sections is illustrated in Figure 1. This study focuses on the rigid pavement test sections.

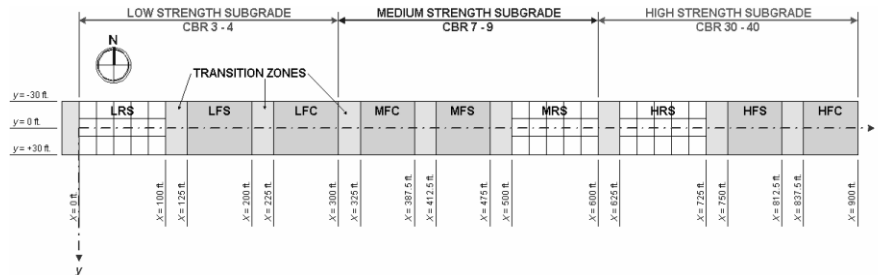


Figure 1: Plan view of FAA's NAPTF (Courtesy of NAPTF website)

Supported on rails on either side, a specially designed test vehicle can apply loads of up to 34,020 kg (75,000 lb) per wheel on two landing gears with up to six wheels per gear and is programmed for a controlled aircraft wander simulation. There are 1,050 sensors installed in the pavement test items recording environmental and dynamic pavement responses in support of NAPTF operations. Data is acquired, processed, stored, and disseminated from the individual sensors using three data collection systems interconnected by wire and wireless local area networks (Tuebert et al., 2002).

During the first series of traffic tests, a six-wheel Boeing 777 (B777) landing gear in one lane and a four-wheel Boeing 747 (B747) landing gear in the other lane were trafficked simultaneously until the test pavements were deemed failed. Non-Destructive Tests (NDTs) using both Falling Weight Deflectometer (FWD) and Heavy Weight Deflectometer (HWD) were conducted to document the uniformity of pavement and subgrade construction as well as to monitor the effect of full-scale trafficking on pavement response and performance over time. This paper focuses on analyzing the rigid pavement non-destructive testing (NDT) data.

NAPTF Rigid Pavement Test Sections

During the first construction cycle (CC-1), a total of forty-five, 6 x 6 m (20 x 20 ft) Portland Cement Concrete (PCC) slabs were placed in three rigid pavement test items as part of the overall test pavement at the NAPTF. Each NAPTF test section is identified using a three-character code, where the first character indicates the subgrade strength (L for low, M for medium, and H for high), the second character indicates the test pavement type (F for flexible and R for rigid), and third character signifies whether the base material is conventional (C) or stabilized (S). The three rigid pavement sections are designated as follows: (a) LRS – rigid pavement with stabilized base over low-strength subgrade, (b) MRS – rigid pavement with stabilized base over medium-strength subgrade, and (c) HRS – rigid pavement with stabilized base over high-strength subgrade.

Cross-sections of the as-constructed PCC test pavements are presented in Figure 2. The items P-501 (PCC), P-306 (Econocrete), and P-154 (granular subbase) are as per standard specifications detailed in the FAA Circular No. AC 150/5370-10A. The P-306 Econocrete is basically a lean mix concrete base course composed of aggregate and cement uniformly blended together and mixed with water. Typically, the 28-day compressive strength is between 5,167 kPa (750 psi) and 8,268 kPa (1,200 psi). A MH-CH soil classification (ASTM Unified Soil Classification System) material known as County Sand and Stone Clay (CSSC) was used for the low-strength subgrade while Dupont Clay (DPC) (CL-CH soil classification) was used for the medium-strength subgrade. The target California Bearing Ratio (CBR) for

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the low-strength subgrade was 4 and 8 for the medium-strength subgrade. The naturally-occurring sandy-soil material with a target CBR of 20 (SW-SM soil classification) at the NAPTF site underlies each subgrade layer. The gradation information as well as the laboratory compaction properties for the subgrade soils and P-154 subbase (called as Grey Quarry Blend Fines) are summarized elsewhere (Hayhoe and Garg, 2001).

LRS		MRS		HRS	
P-501 PCC	280 mm	P-501 PCC	248 mm	P-501 PCC	229 mm
P-306 Econocrete	156 mm	P-306 Econocrete	149 mm	P-306 Econocrete	152 mm
P-154 granular subbase	213 cm	P-154 granular subbase	219 cm	P-154 granular subbase	168 cm
LOW strength controlled subgrade (CBR = 3-4)		MEDIUM strength controlled subgrade (CBR = 7-8)		HIGH strength controlled subgrade (CBR = 30-40)	

Figure 2: Cross-sections of NAPTF rigid test items

Traffic Testing

The NAPTF was dedicated on April, 1999 followed by a 10-month period of verification, shakedown, and pavement response testing. The first series of traffic tests, referred to as CC-1 traffic testing, began in February 2000 and was completed in September 2001. During CC-1 traffic testing, a six-wheel dual-tridem (B777) landing gear, with 1372-mm (54-in) dual spacing and 1448-mm (57-in) tandem spacing was loaded on the north wheel track (LANE 2) while the south side (LANE 5) was loaded with a four-wheel dual-tandem (B747) landing gear having 1118-mm (44-in) dual spacing and 1473-mm (58-in) tandem spacing.

The test machine and the gear configurations used during the first round of traffic testing are shown in Figure 3. The wheel loads were set to 20.4 metric tons (45,000 lbs) each and the tire pressure was 1,295 kPa (188 psi). The traffic speed was 8 km/h (5 mph) throughout the traffic test program. To realistically simulate transverse aircraft movements, a wander pattern consisting of a fixed sequence of 66 vehicle passes (33 traveling in the east direction and 33 traveling in the west direction), arranged in nine equally spaced wander positions (or tracks) at intervals of 260 mm (10.25 in), was used during NAPTF traffic testing.

The wander pattern used during NAPTF traffic testing simulates a normal distribution of aircraft traffic with a standard deviation (σ) of 775 mm (30.5 in) that is typical of multiple gear passes in airport taxiways. Previous full-scale airport pavement traffic tests have used similar wander patterns except that in NAPTF testing, one wander cycle included a larger number of repetitions than were typically used before (Ahlvin et al., 1971).

To minimize the interaction of gear loads at the subgrade level, the two gears moved in phase, with both gears moving forward and backward together. Test vehicle loadings were positioned so that the maximum number of loads in each wander cycle was applied along the edge of the outside lanes. For rigid pavement test sections, failure was defined in terms of

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structural cracking initiating at the joints at the bottom of the PCC layer (McQueen et al., 2002).

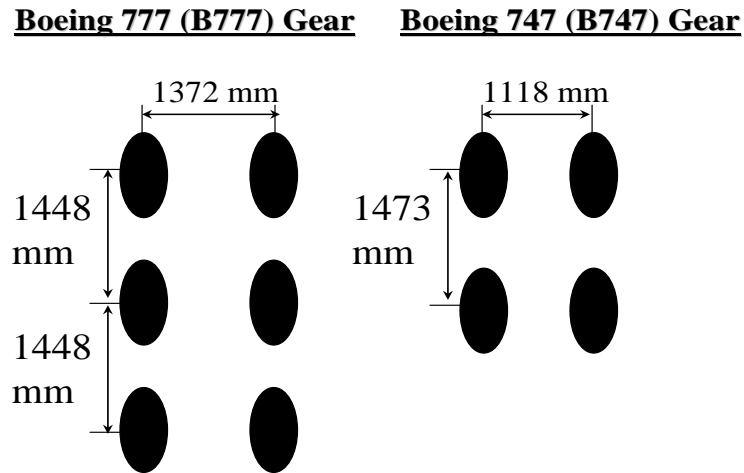


Figure 3: NAPTF traffic test gear configuration details

Non-Destructive Testing

Non-Destructive Tests (NDTs) using both Falling Weight Deflectometer (FWD) and Heavy Weight Deflectometer (HWD) were conducted on NAPTF rigid pavement test sections at various times. The FWD tests are commonly used to assess the structural integrity of highway pavements in a nondestructive manner. For airport pavements, a heavier FWD equipment known as HWD is used to simulate the effect of relatively heavier aircraft wheel loading. The principle of operation is same for both FWD and HWD equipments. There are many advantages to using FWD/HWD tests, in lieu of, or supplement traditional destructive tests for pavement structural evaluation. Most important, is the capability to quickly gather data at several locations while keeping a runway, taxiway, or apron operational during these 2-minute to 3-minute tests, provided the testing is under close contact with Air Traffic Control. Without FWD/HWD, structural data must be obtained from numerous cores, borings, and excavation pits on an existing airport pavement. This can be very disruptive to airport operations. FWD/HWD tests are economical to perform and data can be collected at up to 250 locations per day. The FWD/HWD equipment measures pavement surface deflections from an applied dynamic load that simulates a moving wheel (FAA, 2004).

The deflection data that are collected with the FWD/HWD equipment can provide both qualitative and quantitative data about the strength of a pavement at the time of testing (FAA, 2004). Many studies have addressed the interpretation of pavement deflection measurements as a tool to characterize pavement-subgrade systems, with many of the main findings appearing in American Society for Testing and Materials (ASTM) special technical publications, including those of Bush and Baladi (1989) and Tayabji and Lukanen (2000).

At NAPTF, the FWD tests were conducted to document the uniformity of pavement and subgrade construction. The major objectives of conducting HWD tests at NAPTF were (a) to verify the uniformity of subgrade strength, (b) to determine the joint load transfer ability, (c) to investigate the variation of pavement structural responses with time and temperature, and (d) to further observe pavement distresses during the NAPTF traffic testing (Guo and Marsey, 2001).

For FWD testing, the Engineering & Research International (ERI) Inc. equipment was used. The ERI equipment is a FWD KUAB Model 150 with a segmented 30.5-cm (12-in)

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loading plate and a pulse width of 27-30 msec. The first series of FWD tests were conducted on June 14, 1999 and the subsequent FWD tests were conducted between the end of October, 1999 to the end of January, 2000. Thus, all the FWD tests were conducted prior to the initiation of traffic testing. The FWD tests were performed at nominal force amplitudes of 40-kN (9,000-lb), 60-kN (13,500-lb), 82.3-kN (18,500-lb), and 115.2-kN (25,900-lb).

The HWD tests were conducted using a KUAB 2m HWD device acquired by the FAA. The FAA HWD equipment operates on the principal of dropping weights on a series of hard, rubber buffers separated by a second series of weights and buffers which are connected to a loading plate resting on the pavement surface. These two mass systems results in a consistent and uniform, half-wave sine curve for the loading pulse. The loading plate is segmented into quarters to ensure that the loading force is evenly distributed. Weights and buffers can be added, or removed as necessary, to adjust peak load and loading time. The loading pulse shape is also influenced by the combination of weights and buffers utilized. The drop heights can be adjusted to control the peak load (Guo and Marsey, 2001).

The FAA HWD equipment has seismometers (for measuring deflections) which utilize a spring for reference and Linear Variable Differential Transformer (LVDT) for the sensor. The deflections were measured at offsets of 0-mm (D_0), 305-mm (D_1), 610-mm (D_2), 914-mm (D_3), 1219-mm (D_4), and 1524-mm (D_5) intervals from the center of the load. There is also an additional sensor placed 305-mm (12-in) in front of the load plate (D_{-1}). This arrangement of one sensor (D_{-1}) located in front of the loading plate makes it easy to obtain the load transfer in both directions at a joint. Note that the same sensor locations were used for FWD tests.

The HWD tests were conducted at nominal force amplitudes of 53.4-kN (12,000-lb), 106.7-kN (24,000-lb), and 160-kN (36,000-lb). The FWD/HWD test locations which are of interest in this study (slab center locations) are shown in Figure 4. Note that LANE 1 was trafficked by six-wheel B777 gear loading while LANE 3 was trafficked by four-wheel B747 gear loading. LANE 2 was untrafficked. The same set of FWD/HWD testing points were used in all three rigid pavement sections. Only the following points were tested during the FWD tests conducted on June 14, 1999: West side of the middle transverse joints, North side of the middle longitudinal joints and one of the four corners (Guo and Marsey, 2001). The HWD tests were conducted on the following dates: November 19, 1999; February 11, 2000; March 20, 2000; April 7, 2000; and April 20, 2000. Note that the first set of traffic load repetitions were applied on February 14, 2000.

	Slab 1	Slab 2	Slab 3	Slab 4	Slab 5
LANE 1 B777 Traffic	X	X	X	X	X
LANE 2 C/L	X	X	X	X	X
LANE 3 B747 Traffic	X	X	X	X	X

Figure 4: HWD slab center test locations

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Guo and Marsey (2001) analyzed the results from FWD/HWD tests conducted on rigid test items between summer and winter of 1999 to investigate the slab curling behavior at NAPTF. The analysis of the FWD/HWD test data indicated that the measured deflections at the center of slabs remained almost the same but at the joints and corners, the measurements varied significantly. In wintertime, the deflections at the joints and corners were significantly larger than those measured in the summer. In addition, the joint load transfer capability was lower in winter. A stronger nonlinear relationship between corner deflections and loads was also observed in the wintertime. The test data also showed that the sum of deflections (SD) on two sides of a joint is almost independent to the load transfer capability defined by the ratio between deflections on the unloaded and loaded sides. The current study primarily focuses on analyzing the center slab HWD test data acquired during the traffic testing.

Artificial Neural Network Based Backcalculation Methodology

The elastic modulus of the slab, E , and modulus of subgrade reaction, k , are the two most important backcalculated concrete pavement properties. Over the years, researchers have developed many different methodologies for backcalculation of concrete pavement properties from FWD measurements, including the AREA method for rigid pavements (Ioannides et al., 1989; Ioannides, 1990; Barenberg and Petros, 1991), ILLI-BACK (Ioannides, 1994), graphical solution using ILLI-SLAB (Foxworthy and Darter, 1989), use of regression analysis to solve AREA method for rigid pavements (Hall, 1992; Hall et al., 1996), use of best fit algorithm to find radius of relative stiffness (l) (Hall et al., 1996; Smith et al., 1996), among others.

Based on a backcalculation study of concrete pavement properties using 277 deflection basins obtained from the Denver International Airport (DIA), Rufino et al. (2002) studied the effect of slab modeling (number of layers, interface condition, and model type) as well as effect of different methodologies and sensor configurations on backcalculated pavement properties. It was found that backcalculated slab modulus of elasticity (E) is lower on average when the pavement layers on top of subgrade are bonded versus unbonded interface. Higher backcalculated k -values are obtained when the slab is modeled as plate compared to modeling the slab as elastic layer. Modeling of the slab and base as elastic layers seem to yield more reasonable backcalculated results since the interface bonding condition can be reflected both in the backcalculated slab elastic modulus and subgrade k -value.

In the current study, an Artificial Neural Network (ANN) based methodology was developed for rapid and accurate prediction of backcalculated concrete pavement properties from realistic FWD/HWD deflection basins acquired at the NAPTF during the first series of traffic tests (CC1 traffic testing). ANNs are valuable computational tools that are increasingly being used to solve resource-intensive complex problems as an alternative to using more traditional techniques. Ceylan (2002) employed ANNs in the analysis of concrete pavement systems and developed ANN-based design tools that incorporated the ISLAB 2000 (Tabatabaie and Barenberg, 1978; Khazanovich, 1994; Khazanovich et al., 2000) finite element solutions into routine practical design at several orders of magnitude faster than ISLAB 2000.

Khazanovich and Roesler (1997) developed a program called DIPLOBACK for backcalculation of moduli values of composite pavements based on ANNs. ANNs have also been applied along with dimensional analysis to backcalculate joint properties from FWD testing (Ioannides et al., 1996). The advantage of using ANN and dimensional analysis together is that they both reduce the database size necessary to accurately estimate pavement properties (Rufino et al., 2002). In the development of the new Mechanistic-Empirical

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Pavement Design Guide (MEPDG) for the American Association of State Highway and Transportation Officials (AASHTO), ANNs were recognized as nontraditional, yet very powerful computing techniques and ANN models were used in preparing the concrete pavement analysis package (Khazanovich et al, 2001).

A backpropagation type ANN model was trained in this study with the results from the ISLAB 2000 finite element program and was used as an analysis tool for backcalculating concrete pavement properties from deflection profiles. Backpropagation ANNs are very powerful and versatile networks that can be taught a mapping from one data space to another using example of the mapping to be learned. The term "back-propagation network" actually refers to a multi-layered; feed-forward neural network trained using an error back-propagation algorithm. The learning process performed by this algorithm is called "back-propagation learning" which is mainly an "error minimization technique" (Haykin, 1999). Backpropagation networks excel at data modelling with their superior function approximation capabilities (Haykin, 1999).

A total of 41,106 data vectors generated by modeling slab-on-grade concrete pavement systems using ISLAB 2000 were used for ANN training and testing. Concrete pavements analyzed in this study were represented by a six-slab assembly, each slab having dimensions of 6.1 m by 6.1 m (20 ft by 20 ft). The dense liquid model, proposed by Winkler (1867), was used to characterize the subgrade behavior. To maintain the same level of accuracy in the results from all analyses, a standard ISLAB 2000 finite element mesh was constructed for the slab. This mesh consisted of 10,004 elements with 10,209 nodes.

The ISLAB 2000 solutions database was generated by varying E , k and thickness of PCC (h_{pcc}) over a range of values representative of realistic variations in the field. The E ranged from 6.9 to 103.4 GPa (1,000 to 15,000 ksi); k ranged from 13.6 to 217 MPa/m (50 to 800 psi/in); and h_{pcc} ranged from 152 to 635 mm (6 to 25 in) considering that most design thicknesses would be in this range. Note that the US design approach for concrete pavements limits k to 0.16-0.20 N/mm³. A Poisson's ratio (μ) of 0.15 was assumed for PCC. Thus a total of 41,106 ISLAB 2000 analyses (51 different values of $E \times 31$ different values of $k \times 26$ different values of h_{pcc}) conducted to represent a complete factorial of all the input values.

The ISLAB 2000 solutions database was first portioned to create a training set of 39,106 patterns (95 %) and an independent testing set of 2,000 patterns (5 %) to check the performance of the trained ANN models. A network with two hidden layers was exclusively chosen for the ANN models trained in this study. Satisfactory results were obtained in the previous studies with these types of networks due to their ability to better facilitate the nonlinear functional mapping (Ceylan, 2002; Ceylan et al, 2005).

Separate ANN models were used for predicting each of the backcalculated concrete pavement properties, modulus of subgrade reaction (k) and slab modulus of elasticity (E). Two different ANN backcalculation models were designed by varying the input vectors for each of the outputs. For the first backcalculation model (BCM-6DEF- k), training and testing sets had seven input parameters of D_0 , D_{12} , D_{24} , D_{36} , D_{48} , D_{60} , and h_{pcc} and one output variable of k . The second backcalculation model (BCM-4DEF- k) had the same output variable (k) as in BCM-6DEF, but only five input variables of D_0 , D_{12} , D_{24} , D_{36} , and h_{pcc} . Similar backcalculation models were developed for predicting E .

In the backpropagation type ANNs used in this study, the connection weights are initially selected at random. Inputs from the mapping examples are propagated forward through each layer of the network to emerge as outputs. The errors between those outputs and the correct answers are then propagated backwards through the network and the connection weights are individually adjusted to reduce the error. After many examples (training patterns) have been propagated through the network many times, the mapping function is learned with

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some specified error tolerance. This is called supervised learning because the network has to be shown the correct answers for it to learn (Haykin, 1999; Ceylan et al, 2005).

The 7-60-60-1 architecture (7 inputs [6 deflections + 1 thickness], 60 nodes in the first and second hidden layers, and 1 output node, respectively) was chosen as the best architecture based on its lowest training and testing Mean Squared Errors (MSEs). Figure 5 shows the training and testing MSE progress curves for the 7-60-60-1 network (for predicting k) at the end of 10,000 epochs. Both the training and testing curves for the output are in the same order of magnitude thus depicting proper training. The almost constant MSEs obtained for the last 5,000 epochs also provide a good indication of adequate training of this network.

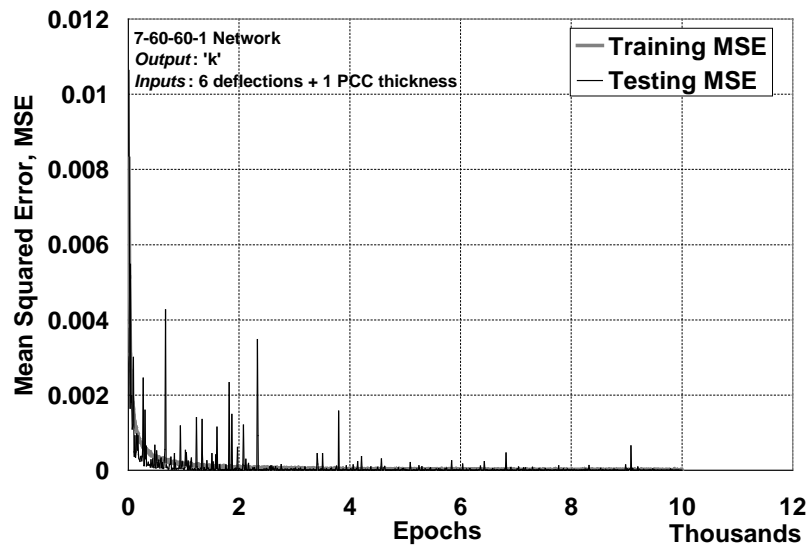


Figure 5: ANN backcalculation model (BCM-6DEF-k) training progress curve

Figure 6 displays the ANN predictions for both k and E using the 7-60-60-1 architectures. All ANN predictions for the 2,000 independent testing sets fall on the line of equality for both k and E, thus indicating a proper training and exceptional performance of the ANN backcalculation models. Average Absolute Errors (AAEs) were calculated as sum of the individual absolute errors divided by the 2,000 independent testing patterns. The AAE for the k-value using the BCM-6DEF-k model was 0.26%. Very similar results were obtained in training and testing the BCM-4DEF-k model, but are not reported here due to space constraints. The AAE for the k-value using the BCM-4DEF-k model was 0.35%. Similarly, the AAEs in predicting E using BCM-6DEF-E and BCM-4DEF-E were 0.32% and 0.34% respectively. Note that all these ANN models were developed for 40-kN (9,000-lb) FWD/HWD loading.

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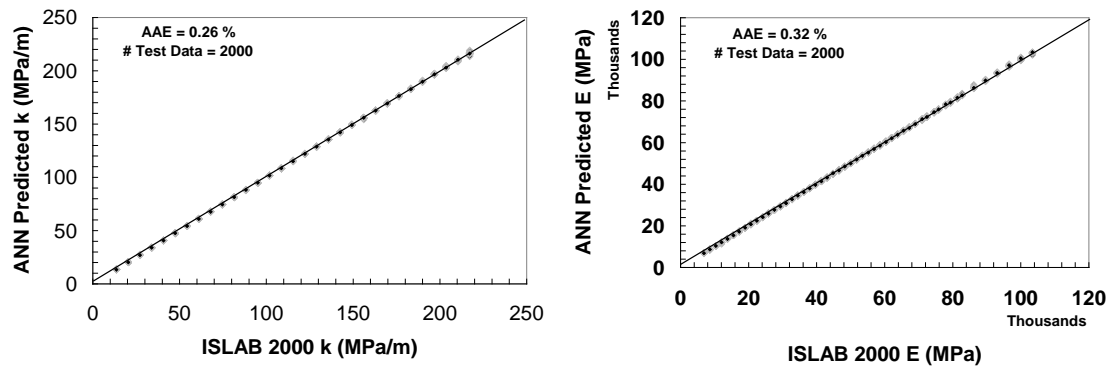


Figure 6: ANN predictions of k and E using 7-60-60-1 architectures

Performance of ANN Backcalculation Models Using NAPTF Field Data

The developed ANN based backcalculation methodology was applied on the actual HWD deflection basins acquired at NAPTF during CC1 traffic testing. The objective was to investigate the effect of time and trafficking on the backcalculated concrete pavement system properties. The backcalculation results predicted using BCM-6DEF are presented here. Since the ANN models have been developed for slab-on-grade pavement systems, the thickness of P-306 Econocrete layer and the PCC slab were combined and transformed into an equivalent thickness based on the stiffness-to-thickness ratios. This transformed thickness was used as the h_{pcc} input for the ANN backcalculation models.

The NAPTF HWD tests were conducted at nominal force amplitudes of 53.4-kN (12,000-lb), 106.7-kN (24,000-lb), and 160-kN (36,000-lb). The HWD tests were conducted on the following dates: November 19, 1999; February 11, 2000; March 20, 2000; April 7, 2000; and April 20, 2000. Note that the first set of traffic load repetitions were applied February 14, 2000.

The changes in 160-kN (36,000-lb) HWD peak center deflections (D_0 s) during NAPTF traffic testing are illustrated in Figures 7 to 9 for all three test items, respectively. Only the results from FWD tests conducted on the slab centers were used in the analysis. The error bars corresponding to plus or minus one standard deviation are also shown. Note that for a normal distribution of values, approximately 68 percent of the values will fall within plus or minus one standard deviation of the mean value. The average pavement temperature at the time of HWD testing as well as the cumulative number of load repetitions (N) are also displayed along with the test dates. Thus, the traffic lane deflection basins reflect the effect of both temperature and trafficking while the untrafficked centerline (C/L) deflections are influenced only by the changes in environment.

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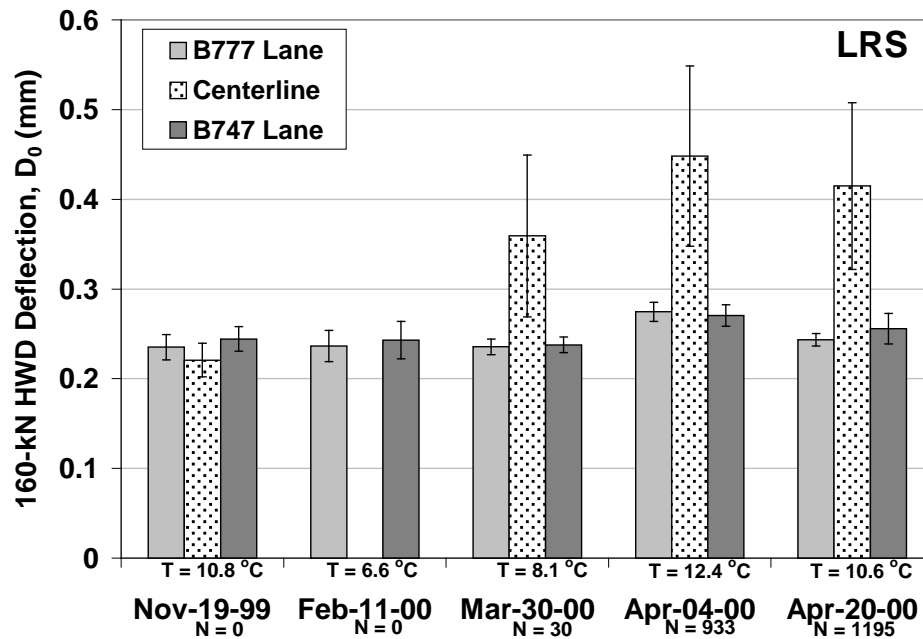


Figure 7: Changes in HWD peak center deflections during traffic testing (LRS)

Guo et al. (2002) analyzed the NAPTF rigid pavement traffic test data. Twenty-eight passes of the NAPTF test vehicle were applied on February 14, 2000. After completion of the initial twenty-eight passes, corner cracks were observed in test items MRS and HRS. No corner cracks were found in LRS after the first 28 passes. However, longitudinal cracks were observed in all slabs in LANE 2 (C/L) of LRS. This is reflected in the C/L HWD peak center deflections as seen in Figure 7. In March 2000, traffic tests were resumed and continued until all the slabs cracked. Ultimately, several slabs were cracked into five and even six pieces (Guo et al., 2002). Corner cracks appeared in LRS during the resumed phase of testing. Trafficking was stopped in HRS test item on March 31, 2000 at 849 passes; in MRS test item on April 6, 2000 at 891 passes; and in LRS on April 10, 2000 at 1,195 passes.

Excessive curling was not expected in the NAPTF rigid pavement test items because the pavements were to be built and loaded indoors. However, significant curling did occur, leading to very early top-down corner cracking. Measurements made after construction showed that the temperature gradients in the slabs were about one tenth what would be expected under typical field conditions and that temperature related curling was insignificant compared to the amounts observed. The large amount of curling observed was eventually considered to have been caused by drying shrinkage and large vertical moisture gradients in the slabs in combination with the large, thin, slabs and, possibly, a mix prone to moisture shrinkage.

Reference to this paper should be made as follows: Bayrak, B., Gopalakrishnan, K., and Ceylan, H. (2006). "Neural Network-Based Backcalculation Models for Non-Destructive Evaluation of Rigid Airfield Pavement Systems," 6th International DUT-Workshop on Fundamental Modeling of Design and Performance of Concrete Pavements, Old-Turnhout, Belgium, September 14-17, 2006.

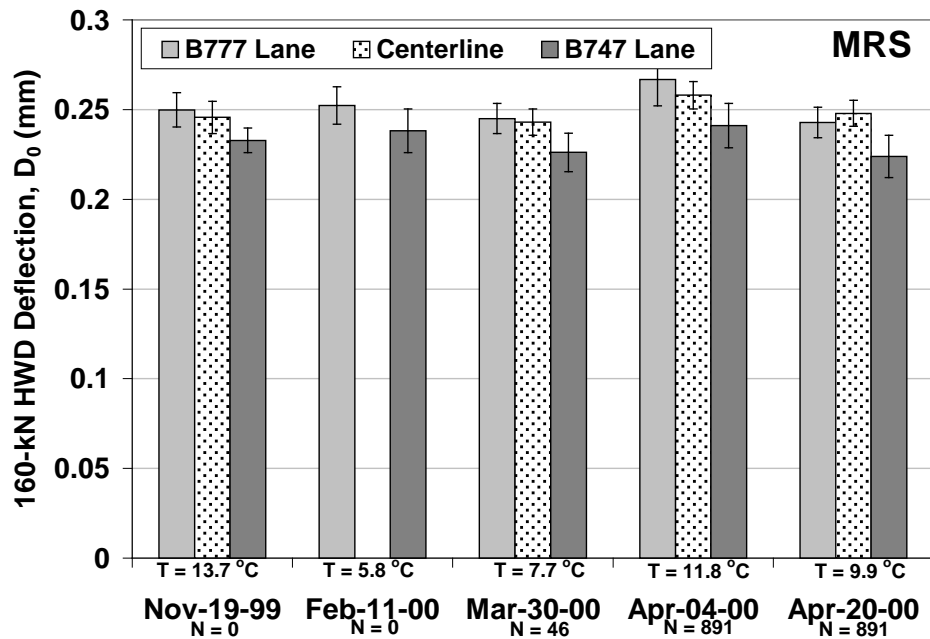


Figure 8: Changes in HWD peak center deflections during traffic testing (MRS)

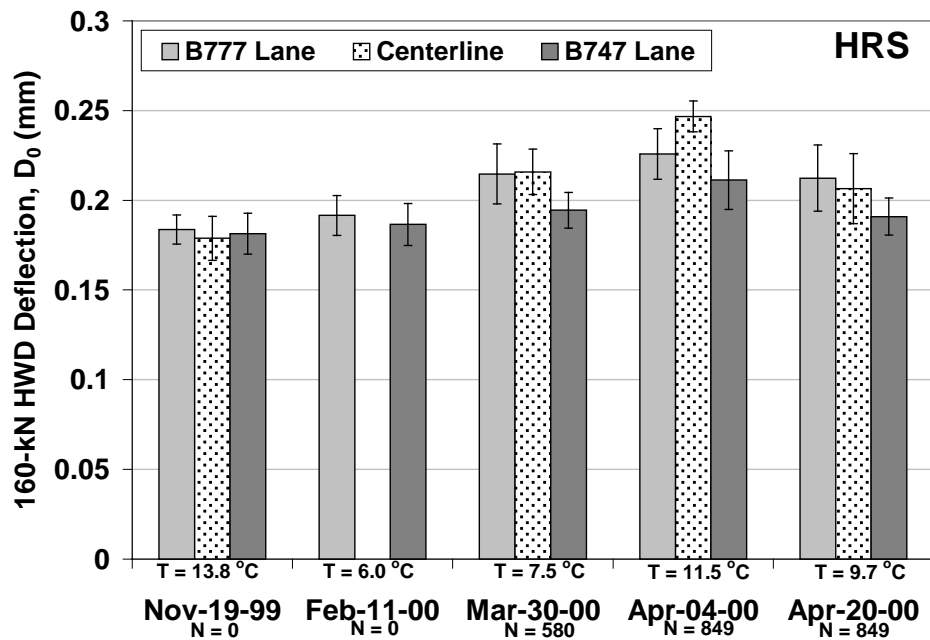


Figure 9: Changes in HWD peak center deflections during traffic testing (HRS)

Crack measurements showed that the HRS slabs exhibited the largest, and the LRS slabs exhibited the smallest, corner cracks. Furthermore, it was found that all pavement slabs were curled up at the corners, with the HRS slabs exhibiting the greatest amount of curling and the LRS slabs exhibiting the least amount of curling. The larger cracked corner areas observed on the HRS slabs are consistent with greater separation of the PCC slab corners from the Econocrete base (Guo et al., 2002). The HWD data collected prior to trafficking indicated significant increase of curling up of the slabs from the summer to the winter of 1999. The HRS test slabs (resting on the high-strength subgrade) were curled more than the LRS test slabs (resting on the low strength of subgrade) (Guo and Marsey, 2001).

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Since the ANN backcalculation models been developed for 40-kN (9,000-lb) loading, the 160-kN (36,000-lb) HWD deflection basins were normalized to a load level of 40-kN (9,000-lb). This is justified since the results from the sensitivity studies conducted at the NAPTF and Denver International Airport (DIA) showed linear FWD/HWD load-deflection behavior for both flexible and rigid pavements and it has been suggested that the amplitude of the FWD/HWD load is not critical provided the generated deflections are within the limits of all deflection sensors (Lee et al., 1997; McQueen et al., 2001; FAA, 2004).

The normalized HWD surface deflections and the equivalent PCC slab thicknesses of the LRS, MRS, and HRS rigid test pavements were used in the BCM-6DEF-k to predict the k values. The ANN predictions of k values for the three test items are shown in Figures 10 to 12, respectively. The variability in predicted k values increased significantly in all three test items as the number of load repetitions increased due to the formation of cracks. Unfortunately, the E values could not be successfully predicted using the ANN models and therefore the results are not reported. Due to the interactions of numerous factors involved, such as slab curling and warping behavior, crack formations, etc., it is hard to distinguish the effect of trafficking alone on the backcalculated concrete pavement properties.

The modulus of subgrade reaction (k) is the stress that will cause one inch of deflection in the underlying soil. Soils such as clay will have a lower k-value compared to cement treated or asphalt treated bases. Research has shown that the value of k depends on certain soil characteristics such as density, moisture, soil texture and other factors that influence the strength of the soils. The k-value of a particular soil will also vary with size of the loaded area and the amount of deflection. The modulus of subgrade reaction is directly proportional to the loaded area and inversely proportional to the deflection. Modulus of subgrade reaction is obtained by conventional plate bearing tests as described in AASHTO T222, correlation with soil properties or other soil tests and also by backcalculation from FWD testing on concrete pavements.

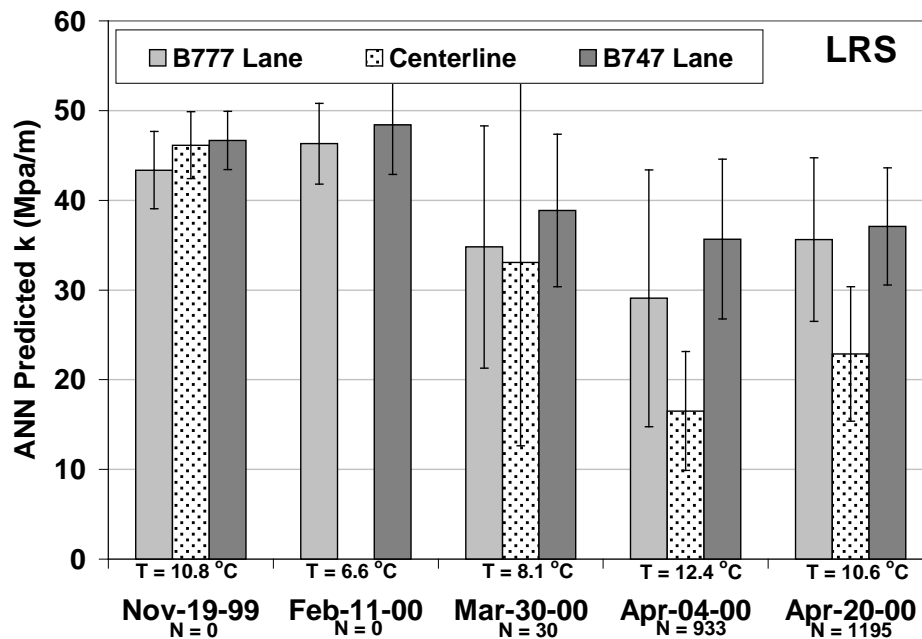


Figure 10: Changes in ANN predicted k values during traffic testing (LRS)

It is observed that the k values for HRS are higher than that accepted for airport pavement design by the current FAA specification (i.e., 135 MPa/m or 500 psi/in). Guo and

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Marsey (2001) pointed out that the objective of the design specification is essentially different from that of the pavement response analysis. The design specification model intends to estimate pavement performance under variable traffic and environmental effects within its service life (usually 20 years for most airport pavements). Whereas, the response model is utilized to predict response of a specific pavement under a well-defined load and/or environmental excitation(s) within a much shorter time span, from a few seconds to a few days. Therefore, for pavement life performance and short time load responses, different values of E and k are reasonable.

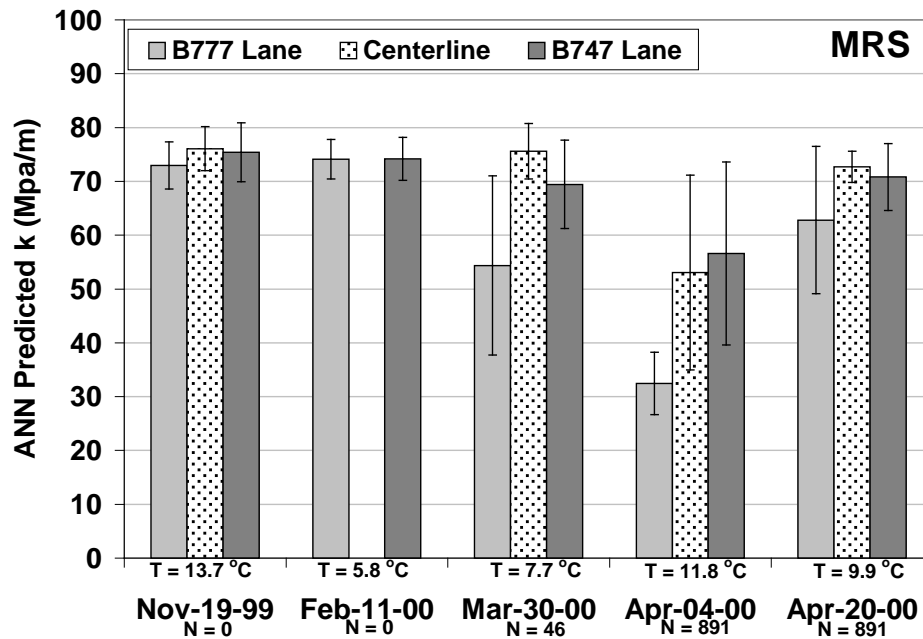


Figure 11: Changes in ANN predicted k values during traffic testing (MRS)

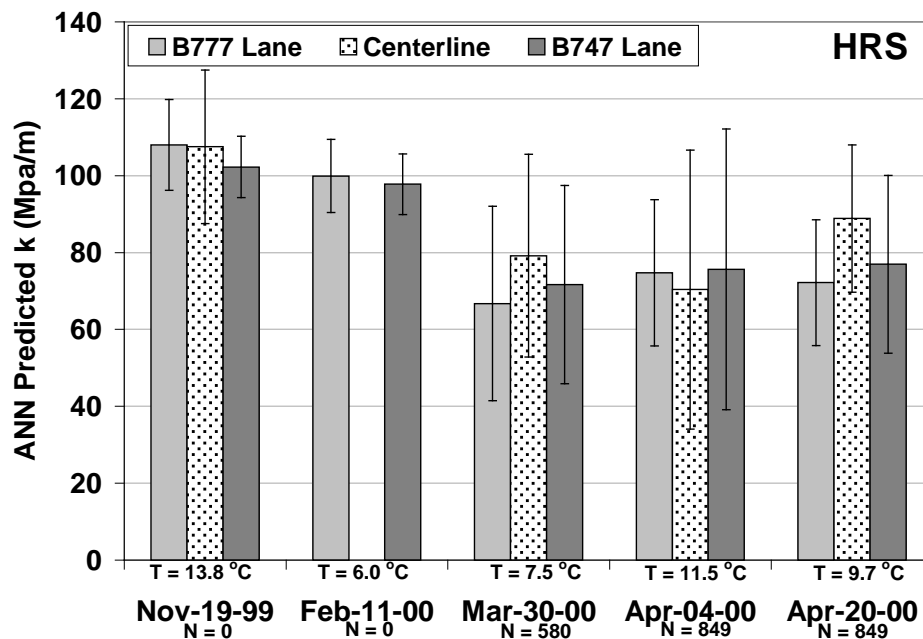


Figure 12: Changes in ANN predicted k values during traffic testing (HRS)

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It is well known that the backcalculated pavement properties are strongly related to the backcalculation model (Rufino et al., 2002). For slab-on-grade systems, Ioannides (1990) proposed a closed-form procedure for backcalculating foundation properties based on principles of dimensional analysis by recognizing the existence of a unique relationship between AREA (Hoffman and Thompson, 1981) and radius of relative stiffness (l_k). Once the radius of relative stiffness is known, Westergaard's (1926) maximum deflection solution for interior loading can be used to backcalculate the subgrade k-value. Once the subgrade properties and radius of relative stiffness are known, the slab modulus of elasticity (E) can also be determined. This approach has been coded in a computer program called ILLI-BACK by Ioannides (1990). The following equations are used:

$$AREA = 6 * \left[1 + 2 \left(\frac{D_{12}}{D_0} \right) + 2 \left(\frac{D_{24}}{D_0} \right) + \left(\frac{D_{36}}{D_0} \right) \right] \quad (1)$$

$$l_k = \left[\frac{\ln \left(\frac{36 - AREA}{1812.279133} \right)}{-2.559340} \right]^{4.387009} \quad (2)$$

$$k = \left(\frac{P}{8D_0 l_k^2} \right) \left\{ 1 + \left(\frac{1}{2\pi} \right) \left[\ln \left(\frac{a}{2l_k} \right) - 0.673 \right] \left(\frac{a}{l_k} \right)^2 \right\} \quad (3)$$

$$E = \left(\frac{12 l_k^4 k (1 - \mu^2)}{h^3} \right) \quad (4)$$

where the deflections are in inches; P is the FWD/HWD load in lbs; a is the radius of load plate (usually 6 inches); and h is the effective slab thickness in inches. Note that the k-value obtained from this backcalculation procedure is the dynamic-k value which is typically twice the k value obtained from plate bearing tests. Using this closed-form backcalculation procedure, the k-values were backcalculated using the same NAPTF HWD deflection data.

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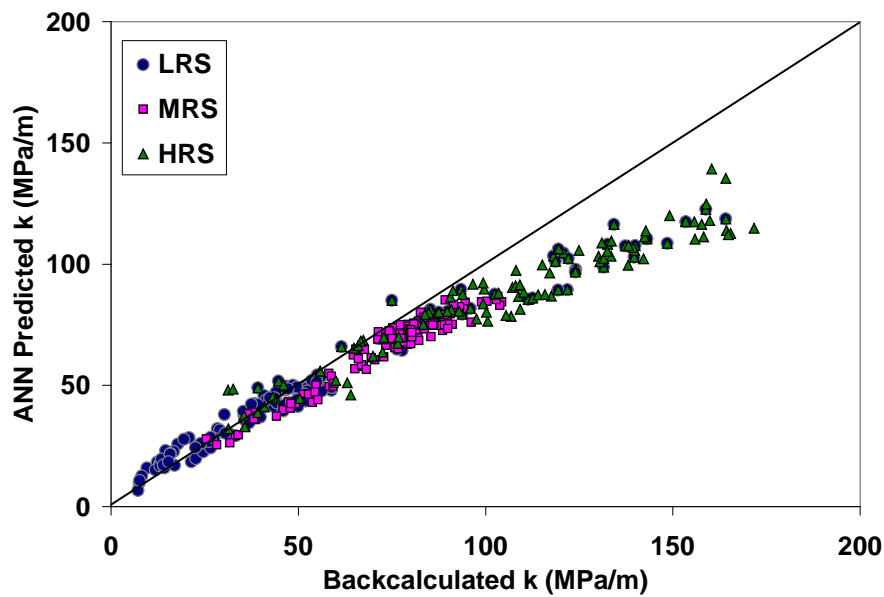


Figure 13: ANN k predictions compared with closed-form solutions

The comparison between the backcalculated k values and ANN predicted k values are presented in Figure 13. There is a good agreement between the two especially for the low-strength and medium-strength rigid test items. At higher k values, the ANN predictions provide conservative estimates compared to the closed-form solutions. The ANN predictions are also in agreement with the results reported by Guo and Marsey (2001) based on HWD test data acquired prior to NAPTF traffic testing.

As mentioned previously, the NAPTF slab modulus of elasticity, E , could not be accurately predicted using the ANN models due to a number of reasons. First, significant curling occurred in NAPTF rigid test items prior to traffic testing, leading to very early top-down corner cracking. Thus, the HWD deflection basins acquired when the slab is in the curled condition can lead to significant error in the backcalculated moduli values. Also, it is well-known that the time of day at which FWD/HWD measurements are made significantly influences the backcalculated slab moduli values even for the same slab location. Also, contrary to what is generally assumed in practice, slab thickness exhibits considerable variability in the field, and this has a large impact on the backcalculated slab stiffness values (Ioannides et al, 1989). During the first cycle of NAPTF traffic testing, thin PCC slabs were required in order to provide design lives within the time available for testing to failure. Also, concrete strengths were also high (about 6.9 MPa [1,000 psi]), another factor leading to thin slabs. All these factors and their interactive effects complicated the backcalculation of E from HWD deflection basins.

Summary and Conclusions

The FAA's NAPTF was constructed to support the development of advanced mechanistic-based airport pavement design procedures. During the first series of traffic tests, a six-wheel Boeing 777 (B777) landing gear in one lane and a four-wheel Boeing 747 (B747) landing gear in the other lane were trafficked simultaneously until the test pavements were deemed failed. Non-destructive tests using both the FWD and HWD were conducted to document the uniformity of pavement and subgrade construction as well as to monitor the effect of full-scale trafficking on pavement response and performance over time.

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ANN-based backcalculation models were developed using the ISLAB 2000 finite-element solutions for rapid and accurate prediction of backcalculated concrete pavement properties from realistic FWD/HWD deflection basins acquired at the NAPTF. It was shown that the modulus of subgrade reaction (k) could be successfully predicted using the ANN models. Unfortunately, the slab modulus of elasticity (E) could not be predicted from the field data although the ANN models showed exceptional performance in predicting the ISLAB 2000 solutions. This is attributed to factors related to field conditions and not to the accuracy of ANN prediction models. The study also illustrated the complexity of backcalculating properties of rigid pavements subjected to full-scale dynamic traffic testing. Apart from other complicating factors such as the slab curling and warping behavior, the test pavements exhibited corner cracks within few passes of traffic loading which further complicated the interpretation of FWD/HWD test results. The ANN predicted k values were in good agreement with those obtained using the closed-form solutions.

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